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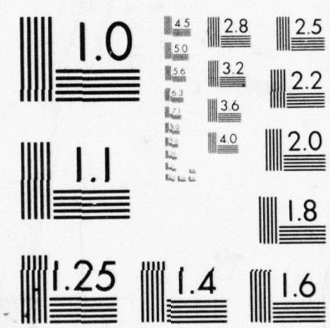

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RETRIEVAL OF BALLISTIC DENSITIES AND LAYER THICKNESSES FROM SATELLITE RADIANCE OBSERVATIONS

Final Technical Report

by

George Ohring,
Principal Investigator

Eliram Broida

Dina Goldberg

June 1977

EUROPEAN RESEARCH OFFICE
United States Army
London, England

GRANT NUMBER DA-ERO-124-74-G0057

Department of Geophysics & Planetary Sciences
Tel-Aviv University, Ramat Aviv, Israel

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The general objective of the work reported on is to develop and evaluate techniques for determining atmospheric parameters in meteorologically 'silent' areas from satellite radiance observations.

A non-statistical method for obtaining ballistic densities directly from satellite radiance observations is derived. The method takes advantage of the fact that both the ballistic density and satellite radiances depend

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upon weighted vertical integrals of the atmospheric temperature. Tests of this method on realistically simulated radiances indicate root-mean-square retrieval errors of 1/4 to 1/3 of the standard deviation of ballistic density for individual months. The method thus appears to be suitable for application to areas of the globe with a paucity of conventional radiosonde observations.

The direct inversion method suggested by Fleming for retrieving layer thicknesses is evaluated on a set of realistically simulated satellite radiances. The results indicate that there is no significant advantage to be gained by using the previous day's temperature profile rather than a monthly mean temperature profile for the geographical area as the standard profile that is required by the method. This result together with the relatively low retrieval errors obtained with the method suggest that it would be quite appropriate for use in meteorologically "silent" areas. The results also indicate that the thickness retrieval error for a deep atmospheric layer appears to be independent of whether the thickness of the deep layer is obtained directly from the satellite radiances or whether the thickness of the deep layer is obtained from the sum of the thicknesses of the sub-layers comprising the deep layer, the thicknesses of the sub-layers being obtained directly from the satellite radiances.

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The general objective of this the work reported on here is to develop and evaluate techniques for determining atmospheric parameters in meteorologically 'silent' areas from satellite radiance observations.

A non-statistical method for obtaining ballistic densities directly from satellite radiance observations is derived. The method takes advantage of the fact that both the ballistic density and the satellite radiances depend upon weighted vertical integrals of the atmospheric temperature. Tests of this method on realistically simulated radiances indicate root-mean-square retrieval errors of 1/4 to 1/3 of the standard deviation of ballistic density for individual months. The method thus appears to be suitable for application to areas of the globe with a paucity of conventional radiosonde observations. Fleming's

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Table of Contents

RETRIEVAL OF BALLISTIC DENSITIES AND LAYER THICKNESSES FROM SATELLITE RADIANCE OBSERVATIONS

1. Introduction

The general objective of the Research Grant was to develop and evaluate techniques for determining atmospheric parameters in meteorologically "silent" areas from satellite based infrared radiance observations. In this Final Report, we review our work on techniques for retrieving from satellite observations two quantities of interest: 1) ballistic densities, and 2) thicknesses of pressure layers.

Both of these quantities are essentially integrated meteorological quantities, that is, it can be shown that they depend upon weighted vertical integrals of the atmospheric temperature. We have chosen such quantities for retrieval from satellite observations because the satellite observations are basically sensitive to the temperatures of broad atmospheric layers rather than to temperatures at individual levels. The deterioration in the accuracy of temperature retrievals as the vertical resolution is improved has been evaluated by Conrath (1972), using the theory developed by Backus and Gilbert for inverse problems related to the solid earth (see, for example, Backus and Gilbert, 1970). In view of these basic theoretical limitations on vertical resolution, we have looked for atmospheric parameters of interest that depend upon vertically integrated temperatures. For such quantities, satellite radiance observations represent an ideal measurement technique. As indicated above, two such quantities are the ballistic density and the thickness of pressure layers.

In order to evaluate the retrieval techniques, we have simulated satellite radiances from sets of observed radiosonde profiles. The simulation procedure is discussed in Section 2. Our method for retrieving ballistic density and the evaluation of the method are discussed in Section 3. In Section 4 we discuss the retrieval method for thickness and the results obtained using this method.

2. Simulation of Radiances

The radiance observed by a satellite radiometer pointing in the nadir direction can be computed from

$$I_i = B_i(T_g)\tau_{ig} + \int_0^{x_o} B_i(T)(d\tau_i/dx)dx \qquad (2.1)$$

where B is the Planck function, T is temperature, τ is the transmittance from the level x to the top of the radiating atmosphere, the index i represents observing wavenumber, the subscript g refers to the surface of the earth, x_o represents the top of the radiating atmosphere, and x, the vertical coordinate is defined by

$$x = -\ln p/p_g \qquad (2.2)$$

The radiances can be simulated with the use of equation (2.1), using numerical integration for evaluating the integral, if the vertical temperature profile and the transmittances are available. We have simulated radiances that would be observed by the six CO_2 band wavenumbers of the NOAA-2 VTPR radiometer (see Table 2.1), using transmittances that have been given by McMillin et al (1973). Radiances were simulated daily for the 12Z temperature profile obtained from the Bet Dagan radiosonde station (latitude 32.0°N, longitude 34.5°E, elevation 30 m) of the Israel Meteorological Service for the months of January, April, July, and October of 1973. These temperature profiles included data from both standard and significant levels. To take advantage of these complete temperature profiles in the numerical integration of equation (2.1), they were combined with the transmittance profiles in the following way. Temperatures were interpolated linearly in $\ln p$ to the 50 levels at which transmittances were available. The transmittances were then interpolated linearly in $\ln p$ to all levels at which temperature observations were available. The combined levels were used in the numerical integration. The level x_o was taken at 0.02 mb pressure and temperatures were extrapolated to this level from the highest level for which a radiosonde temperature was available. The extrapolation was based upon the 30°N standard atmosphere temperature model for the particular month (U.S. Standard Atmosphere Supplements, 1966), the April and October standard temperatures being obtained from the average of the winter and summer values. A similar computation of radiances was performed for the mean temperature profile for each of the four months, based upon the preceding 5 year record of radiosonde observations for Bet Dagan. These sets of radiances represented the standard or climatological radiances.

To simulate the effect of errors due to instrumental effects, clear radiance extraction from cloud contaminated radiances, and other sources, random noise, distributed normally with standard deviations shown in Table 1 (after Fleming, 1972), were added to the computed radiances.

An additional source of error is the uncertainty in the surface temperature T_g, that is used in evaluating the boundary term in equation (2.1) in the inversion procedure. To simulate this error, random noise, normally distributed, with a standard deviation of 1°C was added to the surface temperatures when computing the boundary term in the inversion procedure. The correct surface temperature is used in simulating the radiances using (2.1).

TABLE 2.1

Wavenumbers of NOAA 2 VTPR radiometer and assumed standard deviation of noise (σ_e) at each wavenumber

Wavenumber (cm^{-1})	σ_e [mW/ (cm^2 sec sterdn cm^{-1})]
747.6	0.45
725.9	0.38
709.0	0.33
695.2	0.28
678.0	0.20
668.2	0.20

3. Ballistic Density Retrievals

3.1 Introduction

The ballistic density is essentially a vertically integrated weighted density of the atmosphere and is a quantity that is used, for example, in computing trajectories of vehicles re-entering the atmosphere. Elsberry and Martin (1971) and Elsberry et al (1972) have already shown that this quantity can be retrieved from satellite radiance observations with the use of regression techniques. Such techniques require simultaneous sets of satellite radiance observations and conventional radiosonde observations for their development. For certain regions of the globe it may not be possible to obtain the required sets of data. In the present study, we derive a technique for direct retrieval of ballistic density from satellite radiance observations and test it on a set of realistically simulated satellite radiances.

3.2 Development of Method

The ballistic density may be written as

$$D = \int_{o}^{x_o} F \rho \, dx \qquad (3.1)$$

where D is the ballistic density, F is the ballistic density weighting factor, ρ is the density, and x, the vertical coordinate, is given by

$$x = - \ln(p/p_g) \qquad (3.2)$$

where p is pressure and p_g is the surface pressure. x_o represents a pressure level high in the atmosphere where F is effectively zero. The shape of the weighting function is such that it has a maximum in the middle of the atmosphere and decreases to very small values at the surface and at high altitudes. An example of a ballistic density weighting function is shown in Table 3.1 (after Elsberry and Martin, 1971).

Equation 3.1 shows that the ballistic density is a vertical integral of the weighted density through the depth of the atmosphere. The satellite radiance at each observing wavelength can be expressed as a vertical integral of the weighted temperature through the depth of the atmosphere. The weighting functions for the temperature are narrower than that for the density and are displaced in height depending on the observing wavelength. Since atmospheric densities are related to temperature through the equation of state, it seems reasonable to attempt to obtain the ballistic density directly from radiance observations. Furthermore, since

Table 3.1 Ballistic Density Weighting Function F

Pressure (mb)	F	Pressure (mb)	F
1000		20	
	.1680		.0323
850		10	
	.2421		.0252
700		7	
	.4066		.0193
500		5	
	.4853		.0141
400		3	
	.4637		.0099
300		2	
	.4448		.0061
250		1	
	.3469		.0039
200		0.7	
	.2805		.0027
150		0.5	
	.2114		.0018
100		0.3	
	.1553		.0010
70		0.2	
	.1171		.0007
50		0.1	
	.0836		.0003
30		0.07	
	.0673		
20			

ballistic density is an integral quantity, as is radiance, errors in the determination of ballistic density should be less than the errors in the determination of point values.

Let us define a ballistic density for a standard atmosphere or climatological mean temperature as

$$D_s = \int_0^{x_o} F \rho_s dx \tag{3.3}$$

where ρ_s is the standard or climatological density profile.

The deviation of the actual ballistic density at a particular time from the climatological value can be written as

$$D' = D - D_s = \int_0^{x_o} F (\rho-\rho_s) dx \tag{3.4}$$

With the use of the equation of state $(\rho-\rho_s)$ may be written as

$$(\rho-\rho_s) = \frac{p}{R} \left(\frac{T_s-T}{TT_s} \right) \tag{3.5}$$

where R is the gas constant and T is temperature.

We assume that T in the denominator is equal to T_s. Since the temperature does not vary by more than a few percent about the climatological mean, this assumption introduces, at most, an error of a few percent in the <u>deviation</u> $(\rho-\rho_s)$. And since the ballistic density deviation is a vertical integral of $(\rho-\rho_s)$, errors at one level might cancel errors at another level thus further reducing the final error introduced into the determination of the ballistic density deviation by this assumption.

With this assumption equation (3.4) becomes

$$D' = \int_0^{x_o} \frac{Fp}{RT_s^2} (T_s-T) dx \tag{3.6}$$

Letting

$$W = Fp/RT_s^2 \tag{3.7}$$

we obtain

$$D' = \int_0^{x_o} W(T_s-T) dx \tag{3.8}$$

We turn now to the satellite observed radiance, which may be written as:

$$I_i = B_i(T_g)\tau_{ig} + \int_0^{x_o} B_i(T)(d\tau_i/dx)dx \qquad (3.9)$$

where i is an index representing wavenumber, B is the Planck function, and τ is the transmittance.

The radiance of a standard or climatological atmosphere is:

$$I_{is} = B_i(T_{gs})\tau_{ig} + \int_0^{x_o} B_i(T_s)(d\tau_i/dx)dx \qquad (3.10)$$

Defining (see, for example, Fleming, 1972)

$$r_i = [I_i - B_i(T_g)\tau_{ig}] - [I_{is} - B_i(T_{gs})\tau_{ig}] \qquad (3.11)$$

we obtain

$$r_i = \int_0^{x_o} [B_i(T) - B_i(T_s)](d\tau_i/dx)dx \qquad (3.12)$$

With

$$\Delta B_i = \frac{dB_i(T_s)}{dT} \cdot \Delta T \qquad (3.13)$$

we obtain

$$r_i = \int_0^{x_o} K_i(x)(T-T_s)dx \qquad (3.14)$$

where

$$K_i(x) = \frac{dB_i(T_s)}{dT} \cdot \frac{d\tau_i}{dx}$$

The radiances I_i are observed by the satellite. We assume that the surface temperature T_g, which is needed to evaluate the boundary term in (3.11), can be obtained from either satellite window or conventional observations. Then since the quantities in (3.11) depending on the standard or climatological atmosphere can be computed from (3.10), r_i can be evaluated. Comparison of equations (3.8) and (3.4) shows that they both represent weighted vertical integrals of temperature deviation. Multiplication of equation (3.14) by a set of coefficients c_i and summation over the N observing wavenumbers yields.

$$\sum_{i=1}^{N} c_i r_i = \int_0^{x_o} \left(\sum_{i=1}^{N} c_i K_i\right)(T-T_s)dx \qquad (3.15)$$

Comparison of the right hand sides of (3.8) and (3.15) indicates that if it were possible to find a set of coefficients c_i such that $\sum_{i=1}^{N} c_i K_i$ was exactly equal to W at all values of x then we could obtain (-D'), the negative of the ballistic density deviation, directly and exactly from the left hand side of equation (3.15). In practice such a set of coefficients can not be found but we can find the best approximation to such a set of coefficients by minimizing the form

$$J = \int_0^{x_o} [\sum_{i=1}^{N} c_i K_i - W]^2 dx \tag{3.16}$$

The solution to this minimization problem is

$$\underline{c} = \underline{S}^{-1}\underline{u} \tag{3.17}$$

where

$$S_{ij} = \int_0^{x_o} K_i(x) K_j(x) dx \tag{3.18}$$

and

$$u_i = \int_0^{x_o} W(x) K_i(x) dx \tag{3.19}$$

Thus, once the c_i are determined, the estimated deviation of the ballistic density from the climatological value can be obtained directly from the satellite radiance observations by means of the equation

$$\hat{D}' = -\sum_{i=1}^{N} c_i r_i \tag{3.20}$$

The estimate of the actual ballistic density D is then

$$\hat{D} = D_s + \hat{D}' \tag{3.21}$$

3.3 Results

Tests of the technique were performed with the use of the simulated radiance observations. Radiances were computed daily for the months of January, April, July, and October 1973 from the radiosonde observations at Bet Dagan, Israel. To these radiances was added noise to simulate the effect of instrumental errors, errors due to cloud contaminated radiances, and errors due to the uncertainty in surface temperature, as described in Section 2.

Ballistic densities obtained directly from the simulated radiances with the use of (3.20) and (3.21) were compared to the actual ballistic densities computed from the observed temperature profile with the use of (3.1). Tables 3.2 and 3.3 summarize the results obtained. Table 3.2 refers to retrievals from error free radiances and Table 3.3 to retrievals from radiances with realistic simulation of errors. The standard values of ballistic density, D_s, are computed from the standard temperature profiles, which are based upon the average temperature profile for the month during the five year period 1968-1972.

The results for error free radiances indicate that the ballistic density can be retrieved with an RMS error between 2 and 6, while the standard deviation of ballistic density (σ_D) has values ranging from 14 to 39 (units: $10^{-7} g/cm^3$).

When realistic errors are included in the simulations, the accuracy is degraded, but the results are still extremely encouraging. For example, for the month of January σ_D = 39 while the RMS error in retrieved ballistic density is only 11. The other months also show that the RMS error is only about 1/4 to 1/3 of the standard deviation of the ballistic density.

These results using a direct retrieval method may be compared to the results obtained by Elsberry et al (1972), who used a regression technique. They developed their regression relationships from sets of NIMBUS III SIRS-A clear column radiances and ballistic densities computed from radiosonde data over Eurasia. These regression relationships were then applied to an independent set of satellite observations over Eurasia. The ballistic densities retrieved from the satellite observations were then compared to those computed from radiosonde observations for the same area. In one set of 24 comparisons the RMS error in ballistic density was about 1/3 of the standard deviation of ballistic density for the set; in another set of 16 comparisons, the RMS error was about 1/4 of the standard deviation of ballistic density. These errors are very similar to those that we obtain using the direct method on realistically simulated radiances.

3.4 Conclusions

Based upon simulated observations, it appears that the direct technique for ballistic density determination from satellite radiance observations that has been developed here is a viable alternative to the regression technique used by Elsberry et al (1972). The advantage of a direct method over a regression technique is that there is no requirement for large sets of simultaneous and colocated satellite radiance and conventional radiosonde observations in order to develop the specification equations. On the other hand, the direct technique requires a knowledge of the atmospheric transmittances; to the extent that the true values of the transmittances are uncertain, additional errors will be introduced into the direct technique. It would be of interest to compare both methods using a large set of simultaneous and colocated radiance and radiosonde observations.

TABLE 3.2

Results of tests of direct method for ballistic density determination from satellite observations.

Error free radiances (Units:10^{-7}g/cm^3)

Month	D_s	σ_D	RMSE	$\bar{E}$	$\overline{\lvert E \rvert}$
January	4436	39.1	5.9	3.1	4.4
April	4391	32.4	6.0	2.5	3.5
July	4236	14.5	1.9	-1.3	1.7
October	4357	28.9	4.1	0.6	2.5

D_s = standard ballistic density for the month

σ_D = standard deviation of ballistic density

RMSE = root-mean-square error of ballistic density retrieval

$\bar{E}$ = mean arithmetic error of ballistic density retrieval

$\overline{\lvert E \rvert}$ = mean absolute error of ballistic density retrieval

TABLE 3.3

Results of tests of direct method for ballistic density determination from satellite observations.

Radiances with errors (Units:10^{-7}g/cm^3)

Month	D_s	σ_D	RMSE	$\bar{E}$	$\overline{\lvert E \rvert}$
January	4436	39.1	10.8	2.7	9.2
April	4391	32.4	10.8	- 0.8	8.8
July	4236	14.5	4.6	- 2.5	3.7
October	4357	28.9	7.4	1.0	5.8

Symbols are defined in Table 3.2

4. Layer Thickness Retrievals

4.1 Introduction

The thickness of an atmospheric layer between two prescribed pressure surfaces is proportional to the mean atmospheric temperature of the layer, and, as such, is accessible from satellite radiance observations. Synoptic observations of layer thicknesses are important for operational meteorology since they serve as direct input to a number of numerical prediction models.

There are several ways of obtaining layer thicknesses from satellite radiance observations. The thicknesses can be computed from a temperature profile derived from satellite observations. This procedure has been used by Wilcox and Sanders (1976) in their comparison of thicknesses based upon satellite observations (Nimbus E microwave spectrometer) and those obtained from radiosonde observations. The thickness can also be retrieved directly using statistical techniques (e.g.,Werbowetzki, 1975) or a direct inversion technique (Fleming, 1972) based upon a variant of the Backus and Gilbert (1970) inversion procedure.

In the present study we use the direct inversion technique to answer the following questions:

1) Is there any difference in the thickness retrieval error of a deep layer as derived from the following two methods: a) directly from the radiances, b) as the sum of the thickness of sub-layers that are retrieved directly from the radiances?

2) Is there a reduction in thickness retrieval error when we uses the previous day's temperature profile rather than a climatological mean temperature profile in the procedure that is used to modify the radiative transfer equation in the inversion procedure?

3) What sort of errors can one expect with the use of this direct method for retrieving layer thicknesses?

4.2 Methods

The method described here is based upon Fleming (1972) and further details may be found in his paper. The deviation of a satellite observed radiance from the radiance of a standard or climatological atmosphere, after subtraction of the boundary terms, can be written as (see section 3)

$$r_i = \int_0^{x_o} K_i(x)\Delta T(x)dx \tag{4.1}$$

The deviation of the thickness of a pressure layer bounded by levels x_1 and x_2 from the climatological or standard value for that layer can be written as

$$\Delta H(x_1,x_2) = k \int_{x_1}^{x_2} R(x;x_1,x_2)\Delta T(x)dx \tag{4.2}$$

where k is the gas constant divided by gravity, and $R(x;x_1,x_2)$ is a rectangular function of unit height with cutoffs at x_1 and x_2. If (4.1) is multiplied by k and a set of coefficients c_i, we obtain

$$k \sum_{i=1}^{N} c_i r_i = k \int_{o}^{x_o} \sum_{i=1}^{N} c_i K_i(x)\Delta T(x)dx \tag{4.3}$$

Comparison of (4.2) with (4.3) shows that if we can find a set of coefficients c_i such that R can be approximated by $\sum_{i=1}^{N} c_i K_i(x)$ we can obtain the thickness deviations directly from the satellite observations by means of

$$\Delta H(x_1,x_2) = k \sum_{i=1}^{N} c_i r_i \tag{4.4}$$

The appropriate set of coefficients can be found by minimizing the form

$$\int_{o}^{x_o} [1 - R(x;x_1,x_2)]^2 [\sum_{i=1}^{N} c_i K_i(x) - R(x;x_1,x_2)]^2 dx \tag{4.5}$$

subject to the constraint

$$(x_2 - x_1)^{-1} \int_{x_1}^{x_2} [\sum_{i=1}^{N} c_i K_i(x)]dx = 1 \tag{4.6}$$

(4.5) can be written as the quadratic form

$$s = \underline{c}^T \underline{S}\underline{c} \tag{4.7}$$

The value of s is a measure of the excess width of the approximate rectangular function.

The solution to this minimization problem is

$$c = \underline{S}^{-1}\underline{u}/\underline{u}^T\underline{S}^{-1}\underline{u} \tag{4.8}$$

where
$$S_{ij} = \int_{o}^{x_1} K_i(x)K_j(x)dx + \int_{x_2}^{x_o} K_i(x)K_j(x)dx \tag{4.9}$$

$$u_i = (x_2-x_1)^{-1} \int_{x_1}^{x_2} K_i(x)dx \tag{4.10}$$

The approximate rectangular function

$$\hat{R}(x;x_1,x_2) = \sum_{i=1}^{N} c_i K_i(x) \tag{4.11}$$

is generally wider than the exact R-function. Fleming (1972) shows that one can compensate for this inexactness by correcting the retrieved thickness deviations by multiplying them by the factor a. Thus,

$$\Delta H(x_1,x_2) = a[k \sum_{i=1}^{N} c_i r_i] \tag{4.12}$$

$$a = \int_{x_1}^{x_2} \hat{R}(x;x_1,x_2)dx / \int_{o}^{x_o} \hat{R}(x;x_1;x_2)dx \tag{4.13}$$

If there are observational errors, one would also like to minimize their effects on the thickness errors. This can be done by minimizing the form

$$\sigma^2 = \underline{c}^T \underline{E}\ \underline{c} \tag{4.14}$$

subject to the constraint (4.6). E is a diagonal matrix with diagonal elements ε_i^2, where ε_1 is the RMS error of observation at observing wavelength i.

Minimizing both the thickness error (4.14) and the excess width (4.7) cannot be done, but one can minimize a linear combination of the two by minimizing the form

$$q = \underline{c}^T \underline{Q}\ \underline{c} \tag{4.15}$$

where

$$\underline{Q} = (1-\alpha)\underline{S} + \alpha d\underline{E} \quad , \quad o \leqslant \alpha \leqslant 1 \tag{4.16}$$

where d is a coefficient for matching the physical dimensions of the two terms in $\underline{Q}$. The solution is

$$\underline{c}_\alpha = \underline{Q}^{-1}\underline{u}/\underline{u}^T\underline{Q}^{-1}\underline{u} \tag{4.17}$$

For different values of α one obtains a different solution $\underline{c}_\alpha$, which produces different values of the excess width, s_α, and the thickness error, σ_α^2, according to

$$s_\alpha = \underline{c}_\alpha^T \underline{S}\, \underline{c}_\alpha \text{ and } \sigma_\alpha^2 = \underline{c}_\alpha^T \underline{E}\, \underline{c}_\alpha \tag{4.18}$$

If one plots s versus σ, an L-shaped tradeoff curve is obtained, large excess widths being associated with small thickness errors and vice-versa. Theoretically, the optimum value of α would be at the corner of the L-shaped curve, where $d\sigma/ds = -1$, since at this point both the excess width and thickness error are small.

4.3 Results

Tests of the technique were performed with the use of the simulated radiance observations. Radiances were computed daily for the months of January, April, July, and October 1973 from the radiosonde observations at Bet Dagan, Israel. To these radiances was added noise to simulate the effect of instrumental errors, errors due to cloud contaminated radiances, and errors due to uncertainty in surface temperature, as described in Section 2.

Layer thicknesses retrieved directly from the simulated radiances with the use of the method described in the previous section were compared with those computed from the observed temperature soundings. Two sets of retrievals were made. In the first set, a monthly mean temperature profile based upon the preceding five year record at Bet Dagan was used as the standard or climatological profile in the inversion procedure; in the second set, the previous day's observed temperature profile was used as the standard profile. For each set, thicknesses were retrieved directly from the simulated radiances for four elementary layers, each 200 mb thick, extending from 1000 mb to 200 mb, and for three, deeper, compound layers - 200-600 mb, 600-1000 mb, and 200-1000 mb. Thicknesses for the three compound layers were also derived from summing the retrieved thicknesses of sublayers making up the compound layer. The root-mean-square (RMS) errors of the retrieved thicknesses are shown in Tables 4.1 to 4.4. Also shown in these Tables are values for the variability of the layer thickness with respect to the monthly mean and with respect to the previous day's value.

Inspection of these Tables reveals that there is no significant reduction in retrieval error when the previous day's temperature profile rather than the monthly mean temperature profile is used as the standard profile in the inversion procedure. In fact, there are many cases in which the retrieval errors are smaller when the monthly mean is used as the standard. These results indicate that a monthly mean profile for

Table 4.2 Layer Thickness Retrieval Errors and Layer Thickness Variability in April (gpm)

Layer Boundaries (mb)	Based on Monthly Mean		Based on Previous Day	
	RMS Error	RMS Variability	RMS Error	RMS Variability
Elementary Layers				
1. 200-400	23	31	26	33
2. 400-600	14	33	13	25
3. 600-800	9	27	12	29
4. 800-1000	14	30	17	35
Compound Layers				
5. 200-600	24	56	27	51
6. 600-1000	19	54	23	60
7. 200-1000	28	90	25	90
Summed Layers				
8. 200-600	24	56	28	51
9. 600-800	19	54	23	60
10. 200-1000	25	90	23	90
11. 200-1000	27	90	26	90

Thicknesses (H) in summed layers method are computed from thicknesses of sublayers as follows:

$$H_8 = H_1 + H_2$$

$$H_9 = H_3 + H_4$$

$$H_{10} = H_5 + H_6$$

$$H_{11} = H_1 + H_2 + H_3 + H_4$$

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Table 4.3 Layer Thickness Retrieval Errors and Layer Thickness Variability in July (gpm)

Layer Boundaries (mb)	Based on Monthly Mean		Based on Previous Day	
	RMS Error	RMS Variability	RMS Error	RMS Variability
Elementary Layers				
1. 200-400	15	26	17	21
2. 400-600	12	22	11	15
3. 600-800	10	13	9	9
4. 800-1000	12	17	10	11
Compound Layers				
5. 200-600	13	33	12	27
6. 600-1000	19	28	15	15
7. 200-1000	17	35	16	33
Summed Layers				
8. 200-600	14	33	13	27
9. 600-1000	19	28	15	15
10. 200-1000	16	35	14	33
11. 200-1000	15	35	14	33

Thicknesses (H) in summed layers method are computed from thicknesses of sublayers as follows:

$$H_8 = H_1 + H_2$$

$$H_9 = H_3 + H_4$$

$$H_{10} = H_5 + H_6$$

$$H_{11} = H_1 + H_2 + H_3 + H_4$$

Table 4.4 Layer Thickness Retrieval Errors and Layer Thickness Variability in October (gpm)

Layer Boundaries (mb)	Based on Monthly Mean		Based on Previous Day	
	RMS Error	RMS Variability	RMS Error	RMS Variability
Elementary Layers				
1. 200-400	25	37	22	31
2. 400-600	13	28	12	23
3. 600-800	10	18	12	13
4. 800-1000	14	17	15	16
Compound Layers				
5. 200-600	24	54	23	46
6. 600-1000	19	31	18	22
7. 200-1000	25	68	24	51
Summed Layers				
8. 200-600	25	54	23	46
9. 600-1000	19	31	18	22
10. 200-1000	22	68	20	51
11. 200-1000	23	68	20	51

Thicknesses (H) in summed layers method are computed from thicknesses of sublayers as follows:

$$H_8 = H_1 + H_2$$

$$H_9 = H_3 + H_4$$

$$H_{10} = H_5 + H_6$$

$$H_{11} = H_1 + H_2 + H_3 + H_4$$

a particular geographical area can be used to generate the coefficients required for implementation of the technique during a particular month. This obviates the necessity for recomputing the coefficients on a daily basis, a tremendous savings in computer time. In addition, these results suggest that the technique could work for meteorologically silent areas, the only prerequisite being information on the climatolgoical mean profile for the area, information that could perhaps be obtained from neighboring areas.

The absence of significant improvement in the retrievals when one uses the previous day's profile as the standard rather than using the monthly mean is probably at least partially related to the lack of substantial reduction in thickness variability when one measures the variability with respect to the previous day's thickness rather than with respect to the monthly mean thickness (see RMS variability data in the Tables). The lack of substantial reduction of thickness variability implies that the previous day's temperature profile is not substantially closer to today's temperature profile than is the monthly mean profile. Thus, its use as the standard in the inversion scheme does not significantly affect the retrieval accuracy.

With respect to the question of whether there is any difference in accuracy between the direct method and the summed layers method for determining the thickness of a deep, compound layer - the answer appears to no. There are very small differences between the thickness errors of these two methods - in some cases one method is slightly better, in other cases the other method is better, and in a number of cases the results are identical. These results indicate that the relatively large errors for small layers are essentially random and uncorrelated from one layer to the next such that there is a tendency for cancellation of positive and negative errors when one uses the summed layers method to determine the thickness of a compound layer. For practical applications, these results suggest that if one is interested only in the thickness of several compound layers (e.g., for a two layer numerical forecast model), the thicknesses for the required layers can be obtain by the direct method. If one is interested in the thicknesses of elementary layers as well as compound layers, the thicknesses of the compound layers can be obtained from the summed layers method.

The general RMS error level of the retrieved thicknesses ranges roughly from 10 gpm to 35 gpm, the lower values being associated with the elementary layers and summer conditions (little variability) and the higher values with the compound layers and winter conditions (greater variability). These values may be compared to the accuracies obtainable from the conventional radiosonde observations: ~ 15 gpm RMS error for the 1000-500 mb layer (Wilcox and Sanders, 1976). Thus, the direct method for retrieving layer thicknesses from satellite radiance observations over silent areas appears to be quite viable.

4.4 Conclusions

There appears to be no significant advantage in using the previous day's temperature profile rather than a monthly mean as the standard temperature profile in Fleming's (1972) direct method for retrieving layer thicknesses from satellite radiances.

The thickness retrieval error of a deep atmospheric layer appears to be the same whether one obtains the thickness of the deep layer directly from the satellite radiances or whether one obtains the thickness as the sum of the thicknesses of the sublayers composing the deep layer, the thicknesses of the sublayers being obtained directly from the satellite radiances.

The implications of the above two findings for operational thickness retrievals are discussed in the previous section.

The results obtained in the present study, as well as those of Fleming (1972), suggest that the method should be tested with real satellite observations against other methods currently in use (regression, regression after categorization, from retrieved temperature profiles) to obtain layer thicknesses from satellite radiances.

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Appendix

1. List of publications resulting from research performed under the Grant.

   A paper based upon the work on ballistic density has been submitted to the J. of Appl. Met..

2. List of students who have received support from the Grant and are receiving degrees.

Student	Degree
Eliram Broida	M.Sc.
Dina Goldberg	M.Sc.